國立中央大學九十一學年度轉學生入學試題卷

Linear Algebra

JULY 10, 2002

(10% each)

Let $A = \begin{bmatrix} 5 & 5 & 3 & 3 \\ 5 & 5 & 3 & 3 \\ 3 & 3 & 5 & 5 \\ 3 & 3 & 5 & 5 \end{bmatrix}, b = \begin{bmatrix} 2 \\ 0 \\ -2 \\ 0 \end{bmatrix}$

(a) What is the rank of A?

(b) What is the null space of A (The solution set of $A \cdot x = 0$).

(c) Find an orthonormal basis of the column space of A.

(d) Find an orthonormal basis of the null space of A.

(e) Find the projection p of b onto the column space of A.

(f) Find the solution set of $A \cdot x = p$.

(g) What is the characteristic polynomial of A? minimal polynomial of A?

(h) Find an orthogonal matrix V, a diagonal matrix D such that $A = V \cdot D \cdot V^{-1}$.

(i) Let $B = \exp(A) = \sum_{n=0}^{\infty} \frac{A^n}{n!}$. What is B?

(j) Prove or disprove that if C is a real symmetric matrix, then $\exp(C)$ is positive definite.